

RISK LOAN PORTFOLIO OPTIMIZATION MODEL BASED ON CVAR RISK MEASURE

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Abstract

In order to achieve commercial banks liquidity, safety and profitability objective requirements, loan portfolio risk analysis based optimization decisions are rational allocation of assets. The risk analysis and asset allocation are the key technology of banking and risk management. The aim of this paper, build a loan portfolio optimization model based on risk analysis. Loan portfolio rate of return by using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) constraint optimization decision model reflects the bank's risk tolerance, and the potential loss of direct control of the bank. In this paper, it analyze a general risk management model applied to portfolio problems with VaR and CVaR risk measures by using Using the Lagrangian Algorithm. This paper solves the highly difficult problem by matrix operation method. Therefore, the combination of this paper is easy understanding the portfolio problems with VaR and CVaR risk model is a hyperbola in mean-standard deviation space. It is easy calculation in proposed method.

Key words: Conditional Value-at-Risk; Value-at-Risk; Efficient frontier; Loan portfolio

JEL Classification: B41, C61

I. INTRODUCTION

The Markowitz portfolios (1959) is that one that gives the highest expected return of all feasible portfolios with the same risk. This portfolio is called the mean-variance efficient portfolio. Investors are assumed to seek our maximizing their return while minimizing the risk involved. In ordering to introduce Markowitz's Portfolio theory, Jarrow (1988) first proposed a general equilibrium mode, and then proposed the concept of mean variance efficient frontier. Collinger and Morgan (1993) build bank loan efficient frontier of portfolio model. It is on the basis of portfolio revenue greater than or equal to the target revenue, the establishment of minimum variance loan portfolio allocation model. Altman (1997) presented corporate bond and commercial loan portfolio. It is on the basis of loan portfolio revenue greater than or equal to the target revenue solve the maximum of sharp ratio. Mitra and Khanna (2014) present a simplified perspective of Markowitz's contributions to modern portfolio theory. It is to see the effect of duration of historical data on the risk and return of the portfolio and to see the applicability of risk-reward logic. Chen et al. (2010) introduce the theory and the application of computer program of modern portfolio theory and introduce the mean-variance spanning test which follows directly from the portfolio optimization problem

Conditional value-at-risk (CVaR) is a popular tool for managing risk (Rockafellar and Uryasev, 2000). Value at Risk (VaR) is the methodology used to estimate the market risk to which a bank is exposed, and also for determining, the banks' minimum capital requires to cover this risk. With VaR methodology, not only is exposed risk identified but VaR can also be used as a decision tool to take positions in the market so as to reduce the risk, and, if possible, minimize it. CVaR is the expectation of loss value exceeding the VaR value with the corresponding probability level (Rockafellar and Uryasev, 2000). Rockafellar and Uryasev (2000) showed that CVaR is superior to VaR in optimization applications. Reasons affecting the choice between CVaR and VaR are based on the differences in mathematical properties, stability of statistical estimation, simplicity of optimization procedures, acceptance by regulators, etc. Tongton and Cunbin (2014) study describes a CVaR based investment optimization model, which established electrically portfolio decision making model to optimize the ratio of investment decision making and achieve the maximum yield of the total investment target between the various models of generation. Recently, may Evolutionary Computation (EC) techniques (Beyer, 1996) such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO) (Xu and Cnen, 2006). Delvallet et al., (2008) have been applied to solve combinational optimization problem. In this paper, it analyze a general risk management model applied to portfolio problems with VaR and CVaR risk measures by using Lagrangian Algorithm. We assume that our portfolio is multiple objective problem and the risk factor changes have an elliptical distribution. The class of elliptical distributions is general class of distributions, which contains the normal and the student t-distribution.

II. LOAN PORTFOLIO THEOREM

The theory of VaR and CVaR of risk measurement indicators are based yields lower partial moment (Goh et al., 2012; Lim et al., 2011). The risk lower partial moment measurement theory has obvious advantages than variance theory. For normally distribution random variables, VaR is proportional to the standard deviation. By Rockafellar and Uryasev (2000),

$$VaR = \phi^{-1}(\alpha) \sigma(x) + \mu(x) = \mu(x) + K(\alpha)\sigma(x)$$

Where $k(\alpha) = \sqrt{2} \operatorname{erf}^{-1}(2\alpha - 1)$ and $\operatorname{erf}(z) = (2/\sqrt{\pi}) \int_0^z e^{-t^2} dt$

VaR(x) is nonconvex, discontinuous function. It is a discussion of numerical difficulties of VaR optimization (Rockafellar and Uryasev (2000)). For random variables X, CVaR (x) equals the conditional expectation of X subject to $X \geq VaR(x)$.

CvaR is a coherent risk measure. CVaR is continuous with respect to α . CVaR of a convex combination of random variable CVar ($w_1x_1 + w_2x_2 + \dots + w_nx_n$) is a convex function with respect to (w_1, w_2, \dots, w_n) . CVaR optimization can be reduced to convex programming in some cases to linear programming in this paper using Lagrangian Algorithm.

This paper use the following notations

n: number of available enterprise

r_{it} : the return rate of bank loan of enterprise i and year t, ($i=1, 2, \dots, n$; $t=1, 2, \dots, m$)

w_i : the weight (share) of enterprise i in the portfolio

σ_i^2 : variance of the return on enterprise i

σ_{ij} : covariance between the return on enterprise i and j

ρ_{ij} : correlation between the returns on assets i and j

α : confidence level (example 95% confidence level)

2.1 VaR risk theorem

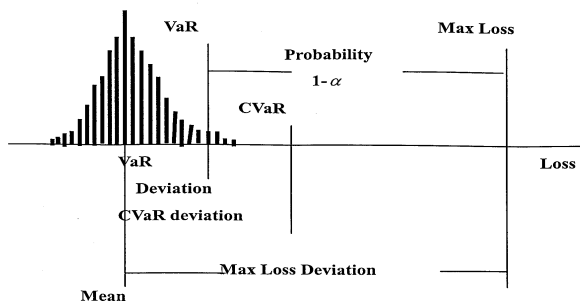


Figure 1: VaR and CVaR deviation
Source: Sarykalin et al. (2008)

Definition 1 (Value-at-Risk)

$$p(L \geq VaR) = 1 - \alpha$$

Then
$$VaR = \mu_{1-\alpha} \tag{1}$$

If $L \sim N(\mu(x), \sigma^2(x))$ and $f(x) = \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $f(x)$ is standard normal probability function, $\phi(x)$ is the cumulative distribution function of x.

$$\begin{aligned}
 CvaR &= E(L|L \geq K^*) = \frac{\int_{k^*}^{\infty} xf(x)dx}{\int_{k^*}^{\infty} f(x)dx} = \frac{1 - \int_{-\infty}^{k^*} xf(x)dx}{1 - \int_{-\infty}^{k^*} f(x)dx} \\
 &= \frac{1}{1-\alpha} f(\mu_{1-\alpha}) \quad (2)
 \end{aligned}$$

The VaR of X with confidence level $\alpha \in [0, 1]$ is

$$p(L \leq VaR) = \alpha \quad (3)$$

For normal distributed random variables, VaR is proportional to the standard deviation

$$p\left(\frac{L - \mu(x)}{\sigma(x)} \leq \frac{VaR - \mu(x)}{\sigma(x)}\right) = \alpha \quad (4)$$

$$\phi\left(\frac{VaR - \mu(x)}{\sigma(x)}\right) = \alpha \quad (5)$$

$$VaR = \phi^{-1}(\alpha) \sigma(x) + \mu(x) \quad (6)$$

Definition 2 (Conditional Value-at-Risk)

$$CVaR = E(L|L \geq \mu_{1-\alpha}) \quad (7)$$

From EQ 2 yields

$$E(L|L \geq \mu_{1-\alpha}) = E\left(\frac{L - \mu(x)}{\sigma(x)} \middle| \frac{L - \mu(x)}{\sigma(x)} \geq \frac{\mu_{1-\alpha} - \mu(x)}{\sigma(x)}\right) = \frac{1}{1-\alpha} f(\mu_{1-\alpha}) \quad (8)$$

By using mathematical statistics

$$E(L|L \geq \mu_{1-\alpha}) = E\left(\frac{L - \mu(x)}{\sigma(x)} \middle| L \geq \mu_{1-\alpha}\right) = \frac{1}{\sigma(x)} E(L|L \geq \mu_{1-\alpha}) - \frac{\mu(x)}{\sigma(x)} \quad (9)$$

From EQ 8, 9 yields.

$$\frac{1}{\sigma(x)} E(L|L \geq \mu_{1-\alpha}) - \frac{\mu(x)}{\sigma(x)} = \frac{1}{1-\alpha} f(\mu_{1-\alpha}) \quad (10)$$

$$CVaR = E(L|L \geq \mu_{1-\alpha}) = \frac{1}{1-\alpha} f(\mu_{1-\alpha})\sigma(x) + \mu(x) \quad (11)$$

2.2 Return rate of bank loan

The return rate of bank loan of enterprise i , at year t is:

$$r_i = \frac{1}{m} \sum_{t=1}^m r_{it} \quad (12)$$

w_i is the weight (share) of enterprise i in the portfolio, the expected return rate of bank loan portfolio is $\mu(x)$

$$\mu(x) = \sum_{i=1}^n r_i w_i \quad (13)$$

$$\sigma_i^2 = \frac{1}{m-1} \sum_{t=1}^m (r_{it} - r_i)^2 \quad (14)$$

$$\sigma_{ij} = \frac{1}{m-1} \sum_{t=1}^m (r_{it} - r_i)(r_{it} - r_j) \quad (15)$$

The risk of bank loan portfolio is $\sigma(x)^2$

$$\sigma(x)^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} = W^T V W \quad (16)$$

Where $w = (w_1, w_2, \dots, w_n)^T$, V denotes as follow:

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2n} \\ \dots & \dots & \dots & \dots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 \end{bmatrix} \quad (17)$$

2.3 CVaR Model

From EQ (11), $(CvaR - \mu(x))/H = CVaR^*$

Where $H = \frac{\varphi(\mu_1 - \alpha)}{1 - \alpha}$

The Markowitz model (mean- variance analysis method): (Huang and Litzenberger, 1988)

$$\begin{aligned} \min \quad & (CVaR^*)^2 = W^T V W \\ \text{S. T} \quad & W^T r = \mu(x) \\ & W^T \mathbf{1} = 1 \end{aligned} \quad (18)$$

Where $\mathbf{1}^T = (1, 1, \dots, 1)$, is a $1 \times n$ column vector of 1's. V is $n \times n$ matrix, and assumes that V is nonsingular matrix and positive definite matrix. Since $\min(x) = \max(-x)$. Using the Lagrangian method with multipliers λ_1 and λ_2 , it is

$$La = -W^T V W + \lambda_1 (W^T r - \mu(x)) + \lambda_2 (W^T \mathbf{1} - 1) \quad (19)$$

It takes the derivate of it with respect to W , λ_1 , λ_2 , set the derivate equation to zero. The derivatives are:

$$\frac{\partial La}{\partial W} = 2VW - \lambda_1 r - \lambda_2 \mathbf{1} = 0 \quad (20)$$

$$\frac{\partial La}{\partial \lambda_1} = r_p - W^T r = 0 \quad (21)$$

$$\frac{\partial La}{\partial \lambda_2} = \mathbf{1} - W^T \mathbf{1} = 0 \quad (22)$$

Solving W from Equation (19) yields.

$$W = \frac{1}{2} V^{-1} (\lambda_1 r + \lambda_2 \mathbf{1}) = \frac{1}{2} V^{-1} [r \ \mathbf{1}] \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \quad (23)$$

Solving and rearranging from Equation (20) yields.

$$W^T [r \ \mathbf{1}] = [\mu(x) \ 1] \quad (24)$$

$$[r \ \mathbf{1}]^T W = \begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} \quad (25)$$

Substituting W from Equation (6) into Equation (8) yields.

$$\begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} = \frac{1}{2} [r \ \mathbf{1}]^T V^{-1} [r \ \mathbf{1}] \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \quad (26)$$

Let

$$\begin{aligned}
 a &= r^T V^{-1} r, \quad b = r^T V^{-1} \mathbf{1}, \quad c = \mathbf{1}^T V^{-1} \mathbf{1}, \\
 d &= ac - b^2 \\
 \mathbf{D} &= \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} r^T V^{-1} r & r^T V^{-1} \mathbf{1} \\ r^T V^{-1} \mathbf{1} & \mathbf{1}^T V^{-1} \mathbf{1} \end{bmatrix} = [r \ \mathbf{1}]^T V^{-1} [r \ \mathbf{1}] \quad (27)
 \end{aligned}$$

Equation (9) can be written as

$$\begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} = \frac{1}{2} \mathbf{D} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \quad (28)$$

Solving the Equation (1) yields.

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = 2\mathbf{D}^{-1} \begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} c\mu(x) - b \\ a - b\mu(x) \end{bmatrix} \quad (30)$$

$$W = V^{-1} [r \ \mathbf{1}] \mathbf{D}^{-1} \begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} \quad (31)$$

Substituting W from Equation (13) into σ_p^2 yields

$$(CVaR^*)^2 = W^T V W = [\mu(x) \ 1] \mathbf{D}^{-1} \begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} = [\mu(x) \ 1] \frac{1}{ac - b^2} \begin{bmatrix} c & -b \\ -b & a \end{bmatrix} \begin{bmatrix} \mu(x) \\ 1 \end{bmatrix} \quad (32)$$

$$(CVaR^*)^2 = \frac{a - 2b\mu(x) + c\mu(x)^2}{ac - b^2} = \frac{1}{d} (c\mu(x)^2 - 2b\mu(x) + a) \quad (33)$$

Equation (15) can be written as:

$$\frac{(CVaR^*)^2}{1/c} - \frac{(\mu(x) - b/c)^2}{d/c^2} = 1 \quad (34)$$

It explains that the portfolio frontier is a hyperbola in mean-standard deviation space.

Equation (34) can be written as:

$$\frac{(CVaR - \mu(x^*))^2}{H^2/c} - \frac{(\mu(x) - b/c)^2}{d/c^2} = 1 \quad (35)$$

Where $\mu(x^*) = \sum_{i=1}^n w_i r_i$

2.4 Global minimum variance portfolio point

Find the minimum variance portfolio of risky assets in this section. The minimum variance portfolio of risky assets denotes as $W_g (\mu(x_m), CVaR^*_m)$. Differentiating Equation (35) with respect to $\mu(x)$ and setting it equation to zero yields. Figure 1 denotes the minimum variance portfolio hyperbola and efficiency Frontier

$$\mu(x_m) = \frac{b}{c}, \quad (CVaR^*)^2 = H^2/c.$$

$$W_g = V^{-1} [r \ \mathbf{1}] \mathbf{D}^{-1} \begin{bmatrix} \mu(x_m) \\ 1 \end{bmatrix} = \frac{V^{-1} [r \ \mathbf{1}]}{ac - b^2} \begin{bmatrix} c & -b \\ -b & a \end{bmatrix} \begin{bmatrix} b/c \\ 1 \end{bmatrix} = \frac{V^{-1} \mathbf{1}}{c} \quad (36)$$

$\mu(x) \%$

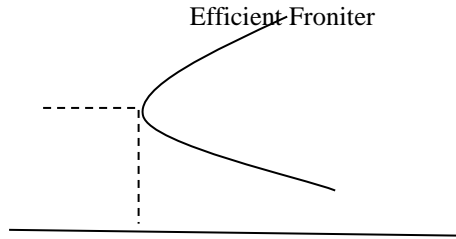


Figure 1 The minimum CVaR* portfolio hyperbola and efficiency Frontier

$$\frac{b}{c} \quad W_g$$

$$\frac{H}{\sqrt{C}} \quad CVaR^*$$

Where $CVaR = H (CVaR^* + \mu(x^*)) = H (CVaR^* + \frac{V^{-1}1}{c} r^T)$

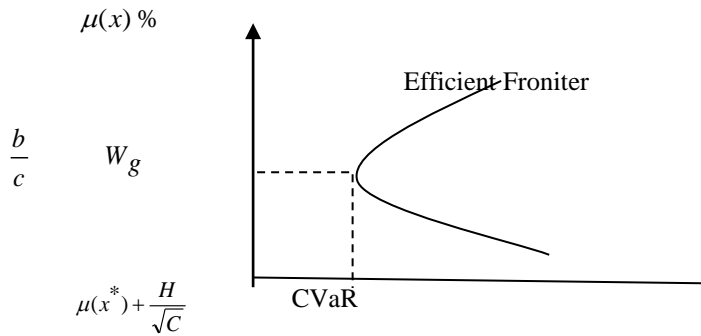


Figure 2 The minimum CVaR portfolio hyperbola and efficiency Frontier

III. THE RELATIONSHIP OF CVaR AND VaR

Equation (6) and (11) solving $\sigma(x)$ yields

$$\frac{CVaR - \mu(x)}{H} = \frac{VaR - \mu(x)}{\phi^{-1}(\alpha)}$$

$$\mu(x) = \frac{\phi^{-1}(\alpha)}{\phi^{-1}(\alpha) - H} CVaR + \frac{H}{H - \phi^{-1}(\alpha)} VaR \quad (37)$$

The intercept of Equation (37) is $\frac{H}{H - \phi^{-1}(\alpha)} VaR$ and the slop is $\frac{\phi^{-1}(\alpha)}{\phi^{-1}(\alpha) - H}$ in $CVaR$ and $\mu(x)$ space.

IV. EMPIRICAL ANALYSIS

4.1 The steps of this proposed method

Portfolio optimization involves a mathematical procedure called quadratic programming problem (OPP). There considered two objectives: to maximize return and minimize risk. The OPP can be solved using constrained optimization techniques involving calculus or by computational algorithm applicable to non-linear programming problem. This paper use matrix operation, it includes matrix inverse, matrix multiplication, and matrix transpose.

Nine enterprises loan returns in this illustration.

Step1: Calculate the efficient frontier inputs

Use Excel function AVERAGE () calculation mean return. Use Excel function STDEVP () calculation standard deviation, for variance it uses VARP (). Use Excel function CORREL () and COVAR () calculation correlation co-efficient and the covariance. Table 1 denoted as 9 enterprise mean return standard deviation and covariance.

Table 1. 9 enterprises' mean return and standard deviation

Enterprises	Mean return (%)	Standard deviation (%)
1	5.32	0.477
2	4.99	0.335
3	4.82	0.367
4	4.29	0.385
5	4.16	0.287
6	1.94	2.182
7	0.05	2.661
8	4.97	0.399
9	5.45	0.461

Step 2: Calculate the efficient frontier by Markowitz portfolios mean- variance analysis method

r represents that 1 x 9 Loan mean returns column vector of 9 enterprise. $V = [\sigma_{ij}]$, represents that 9 x 9 variance- covariance matrix of 9 enterprise returns. W represents that 1 x 9 column vector of the portfolio shares. V^{-1} represents the inverse of matrix V . $\mathbf{1}$ represents that 1 x 9 column vector of 1's. For calculation V and V^{-1} , it uses Excel function COVAR () and MINVERSE ()

From table 1, $r = [0.532, 0.0499, 0.0482, 0.0429, 0.0416, 0.0194, 0.0005, 0.0497, 0.0545]^T$.

For calculation V and V^{-1} , it uses Excel function COVAR () and MINVERSE ()

$$V = \begin{pmatrix} 0.2274 & 0.0315 & 0.0409 & 0.0483 & 0.0488 & 0.5392 & 0.2983 & 0.0960 & 0.1556 \\ 0.0315 & 0.1121 & 0.0208 & -0.0141 & 0.0304 & -0.0371 & 0.4604 & -0.0249 & -0.0016 \\ 0.0409 & 0.0208 & 0.1346 & 0.0549 & 0.0188 & 0.0265 & 0.4343 & 0.0187 & 0.0376 \\ 0.0483 & -0.0141 & 0.0549 & 0.1484 & 0.0568 & 0.0925 & 0.3380 & 0.0502 & 0.0574 \\ 0.0488 & 0.0304 & 0.0188 & 0.568 & 0.0826 & 0.0883 & 0.4388 & 0.0127 & 0.0295 \\ 0.5392 & -0.0371 & 0.0265 & 0.0925 & 0.0883 & 4.7597 & 1.3709 & 0.0867 & 0.1772 \\ 0.2983 & 0.4604 & 0.4343 & 0.3380 & 0.4388 & 1.3079 & 7.0798 & 0.0478 & 0.1038 \\ 0.960 & -0.0249 & 0.0187 & 0.0502 & 0.0127 & 0.0867 & 0.0478 & 0.1595 & 0.1038 \\ 0.1556 & -0.0016 & 0.0376 & 0.0574 & 0.0295 & 0.1772 & 0.0545 & 0.1038 & 0.2127 \end{pmatrix}$$

Calculate the value of a, b, c, and d by using Equation (27)

$a = r^T V^{-1} r$, It uses Excel function MMULT (TRANSPOSE (r), MMULT (V^{-1} , r)) and obtain the value of a

1.046. $b = r^T V^{-1} \mathbf{1}$, It uses Excel function MMULT (TRANSPOSE (r), MMULT (V^{-1} , 1)) and obtain the

value of b 0.4293. $c = \mathbf{1}^T V^{-1} \mathbf{1}$, It uses Excel function MMULT (TRANSPOSE (1), MMULT (V^{-1} , 1)) and obtain the value of c 0.5620.

$$d = ac - b^2 = 0.4036$$

If $\alpha = 0.95$, then $1 - \alpha = 0.05$ and $\mu_{1-\alpha} = 1.65$, $\phi(\mu_{1-\alpha}) = f(\mu_{1-\alpha}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1.65^2}{2}} = 0.1023$

$$H = \frac{\phi(\mu_{1-\alpha})}{1 - \alpha} = \frac{0.123}{0.05} = 2.046$$

From Equation (17), calculation $W_g = \frac{V^{-1}1}{c}$, it uses Excel function MMULT (V^{-1} , 1), and obtain the weight (0, 28.25%, 26.17%, 4.01%, 25.15%, 3.26%, 0, 11.13%, 2.03%). Therefore, the minimum variance portfolio is characterized by an expected return of $\frac{b}{c} = 7.6386\%$ and risk of $\mu(x^*) + \frac{H}{\sqrt{C}} = 6.215\%$ and $\mu(x^*) = 3.486\%$

From Equation (35), the portfolio frontier is a hyperbola in mean-standard deviation space.

$$\frac{(CVaR - 3.4863)^2}{7.4486} - \frac{(\mu(x) - 7.6386)^2}{1.2778} = 1$$

Step 3: calculate global minimum variance portfolio point

The return on the minimum risk portfolio is $\mu(x_m)$ and the minimum standard deviation is $CVaR_m$.

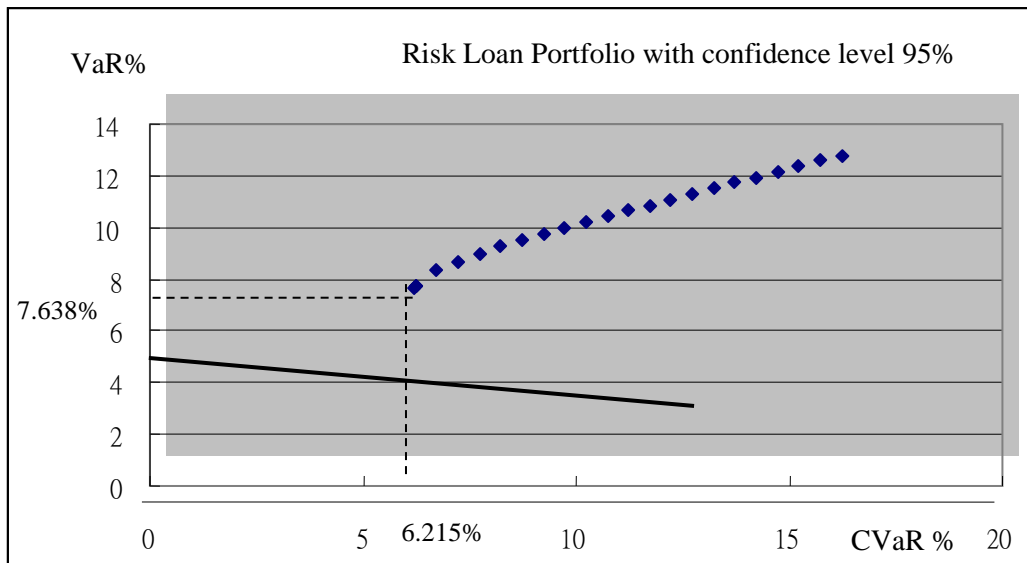
$$\mu(x_m) = \frac{b}{c} = 7.638\%$$

$$CVaR_m = \mu(x^*) + H/\sqrt{C} = 6.215\%$$

Step 4: calculate the relationship of CVaR and VAR

$$\mu(x) = \frac{\phi^{-1}(\alpha)}{\phi^{-1}(\alpha) - H} CVaR + \frac{H}{H - \phi^{-1}(\alpha)} VaR = -4.1667 CVaR + 5.117 VaR$$

Figure 3: The minimum CVaR portf



$$\mu(x) = -4.167 CVaR + 5.117 VaR$$

olio hyperbola and efficiency Frontier

V. CONCLUSION

Under this algorithmic tool, this paper have finished the following works (1) Prove portfolio frontier is a hyperbola in mean-standard deviation space (2) CVaR is a function of α (3) the relationship of VaR with CvaR is linear.

Excel is far from the best program for generating the efficient frontier and is no limited in the number of assets it can handle. It finds that Excel, the computation of the efficient frontier is fairly easy. It finds that this paper is helpful to the correct application of the capital market line based on efficient frontier of portfolio with borrowing and rate, enriching the theory and method of invest management. The further research will focus on performing the calculation of this algorithm tool such as Excel and MATLAB.

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